ASUU JOURNAL OF SCIENCE
A Journal of Research and Development
Vol. 6, No. 1 & 2, December, 2019; pp 24-35

Acceptance Sampling Plan Based on Percentiles for Tsallis q-Exponential Distribution

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Abstract: In this paper, a single acceptance sampling plan based on percentile for Tsallis q-exponential distribution is proposed. The operating characteristics values, the probability of acceptance as well as the minimum number of sample that guaranty the consumer's risk are computed. Comparison with other existing similar plan is made to show the effectiveness of the proposed plan. An illustrative example is given to show the strength of our proposed plan in the manufacturing industry.

Keywords: Acceptance sampling plan, percentiles, Tsallis q-exponential distribution, operating characteristics values

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Introduction

Concerns over variation in the measured qualities of some products especially the life related product led to the development of the statistical quality control. Quality control is the regulatory process through which the measurements of actual quality performance, comparison with other standards are made. Quality control is usually concern with what is called acceptance sampling (Harrison *et. al.*2004). Acceptance sampling plan is an important vital element in the control of quality. It is a system that was developed to protect the consumer from getting unacceptably defective product. A good sampling plan will also protect the producer in the sense that lots produced at permissible levels of quality will have a good chance to be accepted by the plan (Schilling & Neubauer, 2008). Acceptance sampling is therefore an inspection procedure used to determine whether to accept or reject a specific quantity of product produced.

The methods of statistical acceptance sampling is among the interesting and useful applications of modern mathematical statistics (John, 1947). It involves probabilistic distributions and principle of experimental design which provide a ground for these theories to validate basic assumptions in more verifiable and correct inference, than in many biometric and socio-metric applications (Montgomery, 2009). An acceptance sampling plan where a decision of accepting or rejecting the lot is based on one sample only is called single acceptance sampling plan. Operating Characteristics (OC) values are important measure of performance in acceptance sampling plans. These values indicate probability of accepting a lot versus lot fraction defective. An important feature of a sampling plan is how it discriminates between lots of high and low quality. The ability of a sampling plan to discriminate between lots of high and low quality is described by its operating characteristic (OC) values or curve.

Most of the acceptance sampling plans for a truncated life test has considered the determination of sample size as major issue this is due to the use of certain life time distribution. Some works under this direction are the likes of Rosaiah & Kantam (2005) that developed a single acceptance sampling plan based on the inverse Rayleigh distribution mean and Al-Nasser & Obeidat (2019) also considered acceptance sampling plans from a truncated life test based on Tsallis q-exponential distribution (TQED). These authors mentioned above considered developing acceptance sampling using mean of the underline distributions. Whereas Lio *et. al.* (2010), Rao *et. al.* (2016) and Zoramawa *et. al*. (2018) considered developing acceptance sampling using percentiles for the respective distribution.

The authors mentioned above considered the design of acceptance sampling plans based on the population mean under a truncated life test, the plan was argued by Lio *et. al.* (2010) that acceptance sampling plan based on mean samples may not necessarily satisfy the requirement of engineering on the specific percentile of strength or breaking stress. A particular lot may be passed based on the population mean, even though its quality is specified low considering the percentile method. However, a small decrease in the mean with a simultaneous small decrease in the variance can result in a significant downward shift in small percentiles of interest. Rao *et. al.* (2016) stated that engineers nowadays pays more attention to the percentiles of lifetime than the mean life when considering the life time of a product. This is because the percentiles provide more information regarding a life distribution than the mean life does. They concludes that developing acceptance sampling plan based on percentiles of a life distribution can be treated as a generalization of developing acceptance sampling plans based on the mean life of items.

In this paper, we consider the work of Al-Nasser & Obeidat (2019) and modified the TQED to generate the percentile of the distribution. We aim to obtain the minimum sample that will guarantee the life time of the product meet the required specification.

Proposed Acceptance Sampling Plan (ASP) in Tsallis q-Exponential Distribution

Assume that the lifetime of a product follows TQED (Tsallis, 2009) has the following probability density function (pdf) and cumulative distribution function (cdf) respectively:

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Assume that the lifetime of a product follows TQED (Tsallis, 2009) has the following probability density function (pdf) and cumulative distribution function (cdf) respectively:

$$f(t,\lambda,q)=(2-q)\lambda\left[1+(q-1)\lambda t\right]^{\frac{1}{1-q}}, \quad q\neq 1, \tag{1}$$

and $$F(t,\lambda,q)=1-\left[1+(q-1)\lambda t\right]^{\frac{2-q}{1-q}} \tag{2}$$

where $q<2$ is the shape parameter and $\lambda>0$ is the scale parameter.

To develop acceptance sampling plans for the TQED percentiles, the scale parameter in (2) is replaced by $1/\lambda$ and re-define the cdf as:

$$F(t,\lambda,q)=1-\left[1+(q-1)\,t/\lambda\right]^{(2-q)/(1-q)} \tag{3}$$

For given $0<\theta<1$, the $100\theta^{th}$ percentile (or the θ^{th} quantile) is given by

$$t_\theta=\lambda\left[(1-\theta)^{(1-q)/(2-q)}-1\right]/(q-1) \tag{4}$$

The t_θ increases as θ increases, let $\eta=\left[(1-\theta)^{(1-q)/(2-q)}-1\right]/(q-1)$

from (4) $$\lambda=t_\theta/\eta \tag{5}$$

By letting $\delta=t/t_\theta$, then $F(t)$ can be rewritten as follows

$$F(t)=1-\left[1+(q-1)\delta\eta\right]^{\frac{2-q}{1-q}} \tag{6}$$

partially differentiating (6) w.r.t. δ, we have

$$\frac{\partial F(t,\delta)}{\partial\delta}=(2-q)\eta\left[1+(q-1)\delta\eta\right]^{1/(1-q)};\ t>0. \tag{7}$$

Equation (6) and (7) becomes the modified cdf and pdf respectively for percentiles of TQED, where t_θ is the | 10th percentile of the given distribution.

Mostly in life testing acceptance sampling, the procedure is to terminate the testing as soon as the specified limit t is reached, and take a decision to accept or reject the lot considering the probability of acceptance p* and the maximum allowable bad items c. Here, the ASP for percentiles is to obtain the minimum sample size n for the given acceptance number c such that the consumer's risk, the probability of accepting a bad lot does not exceed 1-p* (see Rao et. al. 2012). The bad lot is defined to be the one that fall under 10th percentile in a 100th percentile. However, the probability p* is a confidence level that ensure $t_\theta > t_\theta^0$. Therefore, for a given confidence level, the proposed ASP parameters will be $\left(n, c, \frac{t}{t_\theta^0}\right)$.

Minimum Sample size

As stated by Schilling & Neubauer (2008) for a given confidence p* where p*ϵ (0,1), we reject the lot when $t_\theta > t_\theta^0$. The interest is to find the smallest sample size *n* that will ensure the inequality holds giving the acceptance number *c*. The lot should be sufficient enough to apply the binomial distribution and it must satisfy the following:

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \le 1 - p^* \tag{8}$$

where, $p = F(t,\delta) = F(t) = 1 - \left[1 + (q-1)\delta\eta\right]^{\frac{2-q}{1-q}}$

The above p is the probability of a failure time during time t given a specified percentile of a lifetime t_θ^0 and it depends on the $\delta\eta = t\eta/t_\theta$ since t_θ increases as θ increases.

The smallest sample size n when the distribution follows TQED for percentiles with the same value of the shape parameter as reported in Al-Naseer & Obeidat (2019) one only need to supply the values for the t_θ/t_θ^0 in place of t/μ_0. To save space, the results for the minimum sample size are reported in ***Table 1*** for $t_\theta/t_\theta^0 = 0.7, 0.9, 1.0, 1.5, 2.0, 2.5, 3.0$ and 3.5. and $p^* = 0.75, 0.90, 0.95$ and 0.99. These choices are consistent with Kantam *et al.* (2001), Baklizi and El Masri (2004), Balakrishnan *et al.* (2007) and Zoramawa *et al.* (2018).

Operating Characteristics Values of the sampling plan

The operating characteristics (OC) function describes the chances of a lot passing a sampling inspection; it is denoted by P_a and called probability of acceptance for a given proportion defective (p). Whenever a sampling plan is derived, its description is not complete until its OC function has been described. In our case, the operating characteristics function is the probability of accepting a lot and it given by

$$L(p)=\sum_{i=0}^{c}\binom{n}{i}p^{i}(1-p)^{n-i} \tag{9}$$

where p is a function as in (8) . Note that F(t,λ,q) is represented as a function of $\lambda=t_\theta/\eta$ So using Eq. (9), the OC values can be computed and displayed for any sampling plan. The OC values for the proposed plan is presented in Table 2.

Producer's Risk (α)

The producer's risk is defined as the probability of rejecting the lot by the consumer when in actual sense it is suppose to be accepted i.e. $t_\theta > t_\theta^0$ for a given sampling plan. Our interest is to find the value that will ensure the producer's risk is less than or equal to α for the proposed sampling plan at a specified confidence say p*=0.75. Note that the minimum sample used was computed from Eq. (8) which satisfied the inequality therein Rao *et. al.* (2012).

Consumer's risk (β)

This is the probability for a given (n, c) sampling plan, of accepting a lot with a defective level equal to the LTPD. Here the consumer suffers when this occurs because a lot with unacceptable quality was accepted. The symbol β is commonly used for the type II error and also the range of its values is from 0.01 to 0.2.

Results and Discussion

The authors computed many sample sizes that carter for consumer's and producer's risk specification and presented the results in Table 1 and Table 2. The proposed parameters of percentiles for TQED for various acceptance number of defects i.e. c=0,1,…,10, were found.

If the lifetime of an item follows TQED with shape parameter q=1.2 and a consumer wish to have an item whose lifetime exceed or does not fall below 10th percentile (i.e. t0.1). Now if further the consumer is only willing to accept $\beta = 0.10$ for a 100% life expectancy. For this, we will put on test n = 8 and terminate the inspection once the 3rd failure occurs before the 1000hrs is reached if the life expectance of such item is 1000hrs.

For example, consider an experimenter who wants to establish the true unknown 10th percentile lifetime from simulated data, with c = 2 and p* =0.99, the experiment should make sure the sample of 17 (n = 17) must be considered. If this is done then the pa = 0.8832.

,LOP-0

Comparison between the proposed plan (TQEDP) with similar ASP under TQED

In this section, the proposed plan TQED based on percentiles (TQEDP) is compared with a similar plan TQED based on population mean. The result in Table 3 shows the values of the operating characteristics based

Table 3 : Comparison between OC of TQED and TQEDP

Ratio	0.9	1.50	2.00	2.25	3.00
$OC_{(TQED)}$	0.3322	0.7330	0.8775	0.9345	0.9616
$OC_{(TQEDP)}$	0.8931	0.9353	0.9581	0.9714	0.9850

The proposed plan, indicates that for a given lot with 11 items, the lot is accepted if only fewer than or equal to 2 items fails before time *t*. However, if the lifetime of an item can reach 150% life expectancy then it is assured that the lot will have 0.9353 probability of acceptance, but under a similar plan such an item will have 0.7330 probability of acceptance.

The producer's risk of the proposed plan is 0.0647 which is far below 0.267, on this account, our proposed plan yielded the best minimum sample sizes with the corresponding acceptance numbers that can assure the lifetime of a product does not fall below lower 10^{th} percentile for any given data that follow TQED.

Conclusion

In this paper, the acceptance sampling plan when the lifetime of a product follows Tsallis q-exponential distribution is developed. The minimum sample sizes for the corresponding acceptance number, the operating characteristics values is computed. The procedure for construction of the proposed plan for the percentile is presented.

Based on the results, our plan yielded minimum risk to the producer who aims to minimize the production process cost.

Table 1: Minimum sample sizes necessary to assure the 10th percentile exceed the given default values for the confidence level and corresponding acceptance number *c*

		t_θ/t_θ^0							
P*	C	0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.75	0	3	3	2	2	2	1	1	1
	1	6	5	5	4	3	3	3	2
	2	9	7	7	5	5	4	4	4
	3	12	10	9	7	6	6	5	5
	4	15	12	11	9	8	7	6	6
	5	17	14	13	10	9	8	8	7
	6	20	17	16	12	10	10	9	8
	7	23	19	18	14	12	11	10	10
	8	25	21	20	15	13	12	11	11
	9	28	23	22	17	15	13	13	11
	10	31	26	24	19	16	15	14	13
0.90	0	5	4	3	2	2	2	2	1
	1	8	6	6	4	3	3	3	3
	2	11	9	8	7	6	5	5	4
	3	15	11	10	8	6	6	5	5
	4	18	14	13	10	8	8	6	6
	5	21	16	15	12	9	9	8	8
	6	23	19	18	14	10	10	9	9
	7	26	21	20	16	13	13	10	10
	8	29	24	22	17	15	13	11	11
	9	32	26	25	19	16	15	13	13
	10	35	28	27	20	18	17	14	14

Table 2: Operating characteristics values for the percentiles with plan $\left(n\ ,c\ ,{}^{t_q}\!/\!{}_{t_q^0}\right)$ when c=2

	n	t/θ	1	1.25	1.50	1.75	2.0	2.25	2.50	2.75
0.75	9	0.70	0.9780	0.9876	0.9924	0.9950	0.9965	0.9975	0.9981	0.9986
	7	0.90	0.9803	0.9890	0.9932	0.9955	0.9969	0.9978	0.9983	0.9987
	7	1.00	0.9743	0.9855	0.9910	0.9941	0.9959	0.9970	0.9978	0.9983
	5	1.50	0.9756	0.9862	0.9914	0.9943	0.9961	0.9972	0.9979	0.9984
	5	2.00	0.9510	0.9714	0.9819	0.9878	0.9914	0.9938	0.9953	0.9964
	4	2.25	0.9696	0.9825	0.9891	0.9927	0.9949	0.9963	0.9972	0.9979
	4	3.00	0.9403	0.9645	0.9772	0.9846	0.9891	0.9920	0.9939	0.9953
	4	3.50	0.9160	0.9490	0.9668	0.9772	0.9837	0.9880	0.9909	0.9929
0.90	11	0.70	0.9611	0.9777	0.9861	0.9908	0.9936	0.9953	0.9965	0.9973
	9	0.90	0.9588	0.9763	0.9852	0.9901	0.9931	0.9950	0.9963	0.9971
	8	1.00	0.9619	0.9782	0.9864	0.9909	0.9937	0.9954	0.9966	0.9974
	7	1.50	0.9319	0.9597	0.9743	0.9826	0.9877	0.9910	0.9932	0.9948
	6	2.00	0.9155	0.9493	0.9673	0.9777	0.9841	0.9883	0.9912	0.9932
	5	2.25	0.9357	0.9619	0.9756	0.9835	0.9883	0.9914	0.9935	0.9950
	5	3.00	0.8799	0.9256	0.9510	0.9661	0.9756	0.9819	0.9862	0.9892
	4	3.50	0.9160	0.9490	0.9668	0.9772	0.9837	0.9880	0.9909	0.9929
0.95	13	0.70	0.9394	0.9646	0.9776	0.9850	0.9894	0.9923	0.9942	0.9955
	11	0.90	0.9294	0.9584	0.9735	0.9821	0.9874	0.9908	0.9930	0.9946
	8	1.50	0.9025	0.9411	0.9619	0.9740	0.9815	0.9864	0.9897	0.9920
	6	2.00	0.9155	0.9493	0.9673	0.9777	0.9841	0.9883	0.9912	0.9932
	6	2.25	0.8910	0.9333	0.9565	0.9701	0.9786	0.9841	0.9880	0.9906
	5	3.00	0.8799	0.9256	0.9510	0.9661	0.9756	0.9819	0.9862	0.9892
	5	3.50	0.8364	0.8960	0.9302	0.9510	0.9644	0.9733	0.9795	0.9839
0.99	17	0.70	0.8832	0.9290	0.9539	0.9684	0.9775	0.9834	0.9874	0.9902
	13	0.90	0.8931	0.9353	0.9581	0.9714	0.9796	0.9850	0.9886	0.9912
	12	1.00	0.8891	0.9327	0.9563	0.9701	0.9787	0.9843	0.9881	0.9907
	9	1.50	0.8690	0.9193	0.9470	0.9635	0.9738	0.9806	0.9852	0.9885
	8	2.00	0.8231	0.8879	0.9250	0.9475	0.9619	0.9715	0.9782	0.9829
	7	2.25	0.8378	0.8978	0.9319	0.9525	0.9656	0.9743	0.9803	0.9846
	6	3.00	0.8059	0.8752	0.9155	0.9404	0.9565	0.9673	0.9748	0.9802
	5	3.50	0.8364	0.8960	0.9302	0.9510	0.9644	0.9733	0.9795	0.9839

References

Al-Naseer, A. D., & Obeidat, M. A. (2019). Acceptance sampling plans from truncated life test based on Tsallis q-exponential distribution. Journal of Applied Statistics. doi: 10.1080/02664763.2019.1650254

Baklizi A. and El-Masri A. (2004). Acceptance Sampling based on truncated life tests in the Birnbaum Saunders Model. Risk Analysis 4(6), 1453-1457

Balakrishnan, N., Leiva, V. and Lopez, J. (2007). Acceptance sampling plans from truncated life tests based on the generalized Birnbaum-Saunders distribution. Comm. Statist. Simul. Comp., 36, 643-656

Harrison, M. W., Stephens, K., S. , & Godfrey, A. B. (2004). Modern Methods for Quality Control and Improvement (2nd ed.). Singapore: John Wiley & Son (Asia) Pte. Ltd.

John, H. C. (1947). Acceptance Sampling by variables, with special Reference to the case of Average or Dispersion. Journal of National Bureau of Standards, 39, 271-290.

Kantam, R. R. L., Rosaiah, K. and Rao, G. S. (2001). Acceptance sampling based on life tests: Log-logistic models. J. App. Statist., 28(1) 121 – 128.

Lio, Y. L., Tsai, T.-R., & Wu, S.-J. (2010). Acceptance sampling plans from truncated life tests based on the Birnbaum-Saunders distribution for percentiles. Communications in Statistics -Simulation and Computation, 39, 119-136.

Montgomery, D. C. (2009). Introduction to statistical quality control (6th ed.): John Wiley & Sons.

Rao, G. S., Rosaiah, K., Kaylani, K., & Sivakumar, D. C. U. (2016). A New Acceptance Sampling Plans Based on Percentiles for Odds Exponential Log Logistics Distribution. The Open Statistics & Probability Journal, 7, 45-52.

Rao, G. S., Kantam, R. R. L., Rosaiah, K., & Reddy, J. P. (2012). Acceptance sampling plans for percentiles based on the inverse Rayleigh distribution. Electronic Journal of Applied Statistical Analysis, 5(2), 164-

177. doi: DOI 10.1285/i20705948v5n2p164

Rosaiah, K., & Kantam, R. R. L. (2005). Acceptance Sampling Based on the Inverse Rayleigh Distribution. Economic Quality Control, 20(2), 277 – 286.

Schilling, E. G., & Neubauer, D. V. (2008). Acceptance sampling in quality control (2nd ed.). New York: Taylor and Francis group, CRC Press.

Tsallis, C. (2009). Introduction to Non Extensive Statistical Mechanics. New York: Springer.

Zoramawa, A. B., Gulumbe, S. U., Kantam, R. R. L., & Musa, Y. (2018). Developing double acceptance sampling plans for percentiles based on the inverse Rayleigh distribution. International Journal of Statistics and Applied Mathematics, 3(1), 39-44.

Al-Naseer, A. D., & Obeidat, M. A. (2019). Acceptance sampling plans from truncated life test based on Tsallis q-exponential distribution. Journal of Applied Statistics. doi: 10.1080/02664763.2019.1650254

Baklizi A. and El-Masri A. (2004). Acceptance Sampling based on truncated life tests in the Birnbaum Saunders Model. Risk Analysis 4(6), 1453-1457

Balakrishnan, N., Leiva, V. and Lopez, J. (2007). Acceptance sampling plans from truncated life tests based on the generalized Birnbaum-Saunders distribution. Comm. Statist. Simul. Comp., 36, 643-656

Harrison, M. W., Stephens, K., S. , & Godfrey, A. B. (2004). Modern Methods for Quality Control and Improvement (2nd ed.). Singapore: John Wiley & Son (Asia) Pte. Ltd.

John, H. C. (1947). Acceptance Sampling by variables, with special Reference to the case of Average or Dispersion. Journal of National Bureau of Standards, 39, 271-290.

Kantam, R. R. L., Rosaiah, K. and Rao, G. S. (2001). Acceptance sampling based on life tests: Log-logistic models. J. App. Statist., 28(1) 121 – 128.

Lio, Y. L., Tsai, T.-R., & Wu, S.-J. (2010). Acceptance sampling plans from truncated life tests based on the Birnbaum-Saunders distribution for percentiles. Communications in Statistics -Simulation and Computation, 39, 119-136.

Montgomery, D. C. (2009). Introduction to statistical quality control (6th ed.): John Wiley & Sons.

Rao, G. S., Rosaiah, K., Kaylani, K., & Sivakumar, D. C. U. (2016). A New Acceptance Sampling Plans Based on Percentiles for Odds Exponential Log Logistics Distribution. The Open Statistics & Probability Journal, 7, 45-52.

Rao, G. S., Kantam, R. R. L., Rosaiah, K., & Reddy, J. P. (2012). Acceptance sampling plans for percentiles based on the inverse Rayleigh distribution. Electronic Journal of Applied Statistical Analysis, 5(2), 164-177. doi: DOI 10.1285/i20705948v5n2p164

Rosaiah, K., & Kantam, R. R. L. (2005). Acceptance Sampling Based on the Inverse Rayleigh Distribution. Economic Quality Control, 20(2), 277 – 286.

Schilling, E. G., & Neubauer, D. V. (2008). Acceptance sampling in quality control (2nd ed.). New York: Taylor and Francis group, CRC Press.

Tsallis, C. (2009). Introduction to Non Extensive Statistical Mechanics. New York: Springer.

Zoramawa, A. B., Gulumbe, S. U., Kantam, R. R. L., & Musa, Y. (2018). Developing double acceptance sampling plans for percentiles based on the inverse Rayleigh distribution. International Journal of Statistics and Applied Mathematics, 3(1), 39-44.